# Achieving Better Performance of S-MMA Algorithm in the OFDM Modulation

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### Abstract

Effective algorithms in modern digital communication systems provide a fundamental basis for increasing the efficiency of the application networks which are in many cases neither optimized nor very close to their practical limits. Equalizations are one of the preferred methods for increasing the efficiency of application systems such as orthogonal frequency division multiplexing (OFDM). In this paper, we study the possibility of improving the OFDM modulation employing sliced multi-modulus algorithm (S-MMA) equalization. We compare applying the least mean square (LMS), multi modulus algorithm (MMA) and S-MMA equalizations to the per tone equalization in the OFDM modulation. The paper contribution lies in using the S-MMA technique, for weight adaptation, to decreasing the BER in the OFDM multicarrier modulation. For more efficiency, it is assumed that the channel impulse response is longer than the cyclic prefix (CP) length and as a result, the system will be more efficient but at the expense of the high intersymbol interference (ISI) impairment existing. Both analysis and simulations demonstrate better performance of the S-MMA compared to LMS and MMA algorithms, in standard channels with additive white Gaussian noise (AWGN) and ISI impairment simultanously. Therefore, the S-MMA equalization is a good choice for high speed and real-time applications such as OFDM based systems.

Keywords: Cyclic Prefix, Equalization, ISI, LMS, MMA, OFDM, SMMA.

### 1. Introduction

During recent years, the authors have designed different equalizations for different modulation schemes [1]. Achieving more efficiently orthogonal frequency division multiplexing (OFDM) performance, only by changing the equalization, is the main idea of this paper.

In the most digital communication systems, the inter symbol interference (ISI) occurs due to band-limited channels or multipath propagation. The channel equalization is one of the techniques to decreasing the effect of the ISI [2]. Another way for the cost effective handling of the ISI comes at the expense of the bandwidth efficiency reduction caused by inserting the CP. It is apparent that for more efficiently, the OFDM modulation that can perform well at short CP length is highly desired [3].

In recent years, because of the severity of distortion, the problem of alleviating insufficient-

CP length distortion has received a great deal of attention. Following the early work in [4], where the authors have shorten the channel to reduce the complexity of maximum likelihood sequence estimation (MLSE), the authors in [5] propose a time domain equalizer (TEQ) for digital subscriber line (DSL) systems. In [6], the insufficient-CP distortion was eliminated by a precoder at the transmitter. Moreover, the precoder essentially performs a matrix inversion and thus is prohibitively complex. The work in [6] did not fully take into account the inherent receiver noise and the transmitter power constraint. For some channels, this precoder will result in increasing transmitter power budget and scaling down the precoder coefficients which causes a significant data rate loss. The complexity is then significantly reduced as reported in [5] but the implementation is only applicable for systems with zero CP and still suffers from the power increment problem [7].

The purpose of an equalization algorithm, in single carrier systms, is to make the equalizer match to the inverse of the communication channel impulse response, thus opening the eye of the communication system and allowing for a correct retrieval of the transmitted symbols [8]. However, in the multicarrier systems a TEO is usd in transmitter for shortening the channel impulse response length and also a per-tone equalizer is used in the receiver for decreasing the ISI. There are many different algorithms for updating the tap values of equalizers. The constant modulus algorithm (CMA) [9, 10] is one of these algorithms that have been used for quadrature amplitude modulation (QAM) signals. In order to improve its performance, the authors have been proposed the multi-modulus algorithm (MMA) [8]. In this work, we propose a new MMA algorithm i.e. sliced multi-modulus algorithm (S-MMA) equalization [8], for updating per-tone equalization [11] taps in the OFDM multicarrier modulation. For more qualifying the proposed S-MMA algorithm performance, we test the algorithm with Stanford University Interim (SUI) standard channels in the presence of the ISI, due to an insufficient CP length, and additive white Gaussian noise (AWGN) simultaneously.

The paper is organized as follows. The OFDM modulation description is explained in Section 2 and analysis of the per-tone equalization in the OFDM modulation is explained in Section 3 and analysis of the CP insertion in OFDM modulation is described in Section 4 and S-MMA equalization performance analysis is described in Section 5. In Sections 6 and 7, simulation results and conclusions are presented respectively.

### 2. OFDM modulation description

The basic idea of the OFDM is spliting up a high rate data stream into a number of parallel lower rate data sub-streams, which are transmitted simultaneously over different subcarriers [13]. The OFDM modulation is resistant to multipath interference and frequency selective fading. The OFDM systems also have a relatively simple receiver structure compared to single carrier transmission in frequency selective fading channels [4]. However, the OFDM utilizes the spectrum much more efficiently by spacing the channels much closer together [14]. The OFDM has good performances of anti-ISI, anti-decline, resisting interfere of narrow-band, fitting for asymmetrical transmission and robustness to multipath fading [15]. Because of these advantages, the OFDM has been adopted in both wireless and wired applications in recent years [16].

In the block diagram of an OFDM transmitter, as shown in Fig. 1, a sampled analog signal passes through an analog to digital (A/D) converter and then the resulting bitstream is divided into a number of parallel blocks with a serial to parallel (S/P) converter. These blocks are the input of the constellation mapper, which is basically representing segments of bits as spectral coefficients.



Fig.1. OFDM transmitter block diagram

The resulting sub-channels are orthogonal to each other as long as the CP is longer than the channel impulse response. Otherwise, the system will suffer from insufficient-CP length distortion, which is composed of inter carrier interference (ICI) and the ISI [7, 16].

The channel is modeled so that it adds two forms of interference. i.e. ISI and ICI impairments. It is illustrated that not only ISI but also ICI is caused by the collapse of orthogonality in the received signal. As a result, both the channel identification and equalization become difficult, and the communication performance cannot be guaranteed [17]. The ISI and ICI impairments is removed by turning the linear convolution into a cyclic convolution via insertion a CP at the beginning of each input data stream blocks [18] - [19].

In the receiver, as shown in Fig. 2, the received signal is again broken up into parallel blocks. The CPs are removed and then the FFT of each block is calculated. The equalizer attempts to reduce the ISI in the received signal and maximizes the SNR at the input of the decision circuit. A constellation demapper converts the complex values to a bit stream.

Due to the additive noise, the received constellation points deviate from their location in the original constellation. For recovering the received bitstream, a nearest-neighbor approximation method is computed at each point. The blocks of bits are concatenated back into a single bitstream and then undergoes a D/A convertor and finally back to a sampled analog signal.

## **3.** Analysis of the per tone equalization in the OFDM modulation

With the aid of inverse fast Fourier transform (IFFT) algorithm and appending a CP between the individual blocks at the transmitter and using the fast Fourier transform (FFT) algorithm at the receiver, a broadband frequency-selective channel is converted into a set of parallel flat fading sub-channels or tones [15].

Multicarrier modulation is a powerful technique for providing broadband wireline and wireless communication to customer premises. In wireline applications, multicarrier systems are are used in discrete multitone (DMT) modulation in different variant DSL. Multicarrier is also used in wireless applications such as OFDM defined in IEEE802.11a and HIPERLAN2 standards.



Fig.2. OFDM receiver block diagram

Practical systems use a relatively short cyclic prefix and employ equalization to compensate for the channel effects. The OFDM receiver consists of a real T-tap TEQ, as shown in Fig. 3, for shortening channel impulse response which its outputs are fed to an FFT block that is followed by a complex 1-tap frequency domain equalizer (FEQ) to compensate for the channel amplitude and phase effects. In wireless applications, the goal of TEQ is bit error rate minimization and fast adaptation to nonstationary environment are desired. Per tone equalizer is proposed in 2001 which the structure of a T-tap TEQ in combination with a complex 1-tap FEQ per tone is modified into a structure with a complex T-tap FEQ per tone. As a result, each tone is equalized separately and this leads to a higher bit rate and reduced sensitivity to the synchronization delay[16].

A crucial aspect in this process is that the channel impulse response length may be shorter

or longer than the CP length. In the former, the ISI is removed and only the FEQ is required, whereas for the latter both the frequency and TEQ are needed [11]. The received symbol is the convolution of the transmitted symbol and the channel impulse response h=[h0,..., hL], plus additive noise.

For inserting the CP to each symbol, we use the P matrix as bellow

(1)

$$\mathbf{P} = \left[\frac{\mathbf{0} \mid \mathbf{I}_{c}}{\mathbf{I}_{N}}\right]$$

which I and 0 matrices are 'identity' and 'zero' matrices respectively and their indexes show the size of the matrices [11]. Considering three successesive OFDM symbols  $X_{EN}^{(c)}$  for time C = k - 1, k, k + 1, the received signal will be [11]



(2)

where INFFT is an N×N IFFT matrix that modulates the input symbols. Also O(1) and O(2) are zero matrices of size

 $(N+T-1)\times(N+v-T+1-L+v)$  and  $(N+T-1)\times(N+v-K)$  respectively.  $\overline{\mathbf{h}} = [h_L,...,h_0,...,h_{-k}]$  is the channel impulse response in reverse order, yi and ni for i= 1, 2,..., N, are



Fig. 3. T-tap TEQ equalizer for OFDM receiver

the ith component of the received and noise vectors respectively. The conventional receiver with TEQ is based on the following operation

$$\begin{bmatrix} \mathbf{Z}^{(k)} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{Z}_{N}^{(k)} \end{bmatrix} = \begin{bmatrix} D_{1} & \mathbf{O} & \cdot & \cdot & \cdot & \mathbf{O} \\ \mathbf{O} & \cdot & \cdot & \cdot & \cdot & \cdot & \mathbf{O} \\ \vdots & \cdot & \cdot & \cdot & \cdot & \cdot & \mathbf{O} \\ \vdots & \cdot & \cdot & \cdot & \cdot & \cdot & \vdots \\ \vdots & \cdot & \cdot & \cdot & \cdot & \cdot & \vdots \\ \vdots & \cdot & \cdot & \cdot & \cdot & \cdot & \vdots \\ \mathbf{O} & \cdots & \cdots & \cdots & D_{N} \end{bmatrix} \cdot \overrightarrow{F_{NFFT}} \cdot (\mathbf{Y} \cdot \mathbf{W})$$

(3)

where Y matrix is a Toeplitz matrix and is defined as

$$\mathbf{Y} = \begin{bmatrix} y_{k,s+\nu+1} & y_{k,s+\nu} & \cdots & y_{k,s+\nu-T+2} \\ y_{k,s+\nu+2} & y_{k,s+\nu+1} & \cdots & y_{k,s+\nu-T+3} \\ \vdots & & & & \\ \vdots & & & & \\ \vdots & & & & \\ y_{(k+1),s} & y_{(k+1),s-1} & \cdots & y_{(k+1),s-T+1} \end{bmatrix}_{N \times T}$$

(4)

The main idea of per tone equalization is transferring TEQ from time domain to frequency domain which is performed the following permutation for each tone i

$$Z_{i}^{(k)} = D_{i}.row_{i}(F_{NFFT})(\mathbf{Y}.\mathbf{W}) = row_{i}(\underbrace{F_{NFFT}}_{TFFT}.\mathbf{Y}).\underbrace{\mathbf{W}.D_{i}}_{\mathbf{W}_{i}}$$
(5)

In this work, the per-tone equalizer, which is shown in Fig. 4, is used in the OFDM receiver part and instead of the well-known algorithms such as the LMS, the MMA and S-MMA algorithms are used for updating the per-tone equalizer taps. The CP block through a serial to parallel convertor, as shown in Fig. 4, is removed. Each CP bit and the corresponding bit are compared for determining the channel effects on the data bit stream [11, 16]. In this figure  $\Delta = N+v$ is the OFDM symbol length, where N is symbol size and v is the CP length. Also Vi,l is the coefficient of the per-tone equalizer and finally ↓N+v denotes down-sampling with period of N+v samples. The modified per tone equalizer is defined as  $\mathbf{V}_i = [\mathbf{v}_{i,o}, \dots, \mathbf{v}_{i,T-1}]^T$  and therefore Wi coefficients will be computed as



Fig. 4. Per-tone equalizer for OFDM receiver

# 4. Analysis of the CP Insertion in the OFDM Modulation

For considering the effects of the CP insertion, first we define the OFDM symbol in the baseband as

$$x_{l}[n] = \sum_{i=-N/2}^{N/2-1} a_{i,l} e^{j\frac{2\pi}{N}in} s[n]$$
(9)

whereai, l denotes the complex symbol modulating of the ith subcarrier. The s[n] is a time rectangular window function in the interval [0,M], where M is the OFDM symbol period, and N is the number of subcarriers.

After the CP insertion with the length of Ng, the lth discrete time domain of the OFDM support  $\tilde{x}_{i}[n]$  will be

symbol 
$$x_{l}[n] = \sum_{i=-N/2}^{N/2-1} a_{i,l} e^{j\frac{2\pi}{N}i(n-N_{g})} s[n]$$
 (10)

The channel impulse response with the channel tap weight coefficients hl and the symbol period T is

$$h(t) = \sum_{l=0}^{L} h_l \delta(t - \frac{lT}{N})$$
(11)

Therefore, the output of the OFDM transmitter will be

$$x(t) = \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i,m} e^{j\frac{2\pi i}{T}(t-mT)} s(t-mT)$$
(12)

The received baseband signal at the input of the OFDM receiver will be as the linear convolution of the transmitted signal and the channel impulse response as bellow

$$u(t) = h(t) * x(t) = \sum_{l=0}^{L} h_l x(t - \frac{lT}{N})$$
  
=  $\sum_{l=0}^{L} h_l \sum_m \sum_{i=-N/2}^{N/2-1} a_{i,m} e^{j\frac{2\pi i}{T}(t - \frac{lT}{N} - mT)} s(t - \frac{lT}{N} - mT)$  (13)

The k'th subcarrier of the m'th demodulated OFDM symbol will be



Fig. 5. Insertion of GI

$$u_{k',m'} = \frac{1}{N} \sum_{l=0}^{L} h_l \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i,m} e^{-j\frac{2\pi i}{N}} \\ \times \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}((i-k')n + (m'-m)(i-k')N)} \\ \times s(\frac{nT}{N} - \frac{lT}{N} + (m'-m)T)$$
(14)

It was proved [12] that the Eq. (14) will be rewriten as

$$u_{k',m'} = \sum_{l=0}^{L} h_l e^{-j\frac{2\pi k'l}{N}} (1 - \frac{l}{N}) a_{k',m'} + \frac{1}{N} \sum_{\substack{i=-N/2\\ i\neq k'}}^{N/2-1} a_{i,m'} \sum_{l=0}^{L} h_l e^{-j\frac{2\pi l}{N}} \sum_{n=l}^{N-1} e^{-j\frac{2\pi}{N}(i-k')n} + \frac{1}{N} \sum_{\substack{i=-N/2\\ i=-N/2}}^{N/2-1} a_{i,m'-1} \sum_{l=0}^{L} h_l e^{-j\frac{2\pi l}{N}} \sum_{n=0}^{l-1} e^{-j\frac{2\pi}{N}(i-k')(n+N)}$$
(15)

In Eq. (15), the first term is the desired data, the second term represents the ICI caused by the other subcarriers belonging to the current OFDM symbol. Finally, the third term represents the ISI caused by the subcarriers of the previous OFDM symbol [12].

The ISI can be avoided by the insertion of a guard interval (GI) at the beginning of each OFDM symbol. The GI, with Ng length, should be longer than the maximum possible of the channel impulse response length. In order to avoid ICI, the last part of the OFDM symbol can be added to the beginning of the symbol, as shown in Fig. 5. This part is called the CP. After inserting the CP, the Eq. (15) will change to

$$u_{k',m'} = \sum_{l=0}^{L} h_l e^{-j\frac{2\pi k' l}{N}} a_{k',m'}$$
(16)

which contains only the desired symbol, free of the ICI and ISI impairments. Therefore, by inserting a GI longer than the maximum delay spread of the channel and by cyclically extending the OFDM symbol over the GI, both the ICI and ISI eliminated completely and the channel appears to be flat fading for each subcarrier [11, 12].

## 5. S-MMA Equalization Performance Analysis

The relation between the transmitted symbol x(m) and the received signal u(m), as shown in Fig. 6, will be

$$u(m) = \sum_{i=0}^{L-1} h_i x(m-i) + n(m)$$
(17)

where hi is the ith tap of the channel impulse response with length L and n denotes the AWGN noise.

$$a(\mathbf{m})$$
  $u(\mathbf{m})$   $y(\mathbf{m})$ 



Fig.6 .Blind equalizer block diagram

Least mean square (LMS) is a well-known algorithm for updating different adaptive filter. For the LMS algorithm, the estimation error is expressed as

$$e(m) = x(m) - y(m) = x(m) - \hat{\mathbf{w}}^{H}(m)\mathbf{u}(m)$$
(18)

where  $\hat{\mathbf{w}}^{(m)}$  is the estimation of the tap-weight vector at iteration m and H denotes the Hermitian operator. In this case, the LMS cost function will be

$$J = J_{\min} + 2 \operatorname{Re} \left\{ E\left(e_{o}^{*}(m)\boldsymbol{\varepsilon}_{o}^{H}(m)\mathbf{u}(m)\right) \right\} + E\left(\boldsymbol{\varepsilon}_{o}^{H}(m)\mathbf{u}(m)\mathbf{u}^{H}(m)\boldsymbol{\varepsilon}_{o}(m)\right)$$
(19)

where  $\int^{min}$  is the minimum mean-square error of Wiener filter and  $\varepsilon_o(m)$  is the zero-order weight-error vector of the LMS filter [20]. The LMS tap updating algorithm is

$$\mathbf{w}(m+1) = \mathbf{w}(m) + \mu \mathbf{u}(m)e^{-m}(m)$$
(20)

where w(m) is the tap-weight vector at iteration m and  $\mu$  is the step size and u(m) is the received signal vector or the LMS input vector [20].

The CMA is a special case of Godard's family of blind equalization algorithms [21]. Its cost function is only amplitude-dependent, and knowledge about the signal constellation is dismissed. For signal constellation which all signal points have the same magnitude, the performance of CMA is reasonable [8]. Many MMA have been presented in the past to overcome the misadjustment caused by the CMA. Some of these MMA schemes, specifically for QAM constellations, fix the phase offset error without needing any rotator at the end of the equalizer stage. The MMA minimizes the dispersion of real and imaginary parts, yR and yI, of y(n) separately [22]. The MMA, unlike the CMA, ignores the cross term yRyI between the inphase and quadrature components. As a result, the MMA cost function is not a two-dimensional cost function and it is pseudo two-dimensional because it contains yR(n) and yI(n) only [8]. The MMA cost function and its parameters are given as

$$J = E\left\{ \left( y_{R}^{2}(m) - R_{R} \right)^{2} + \left( y_{I}^{2}(m) - R_{I} \right)^{2} \right\}$$
(21)

$$R_{R} = \frac{E[x_{R}^{4}]}{E[x_{R}^{2}]} , \qquad R_{I} = \frac{E[x_{I}^{4}]}{E[x_{I}^{2}]}$$
(22)

which yR(m) and yI(m) are the real and imaginary parts of y(m). The corresponding MMA tap updating algorithm is

$$\mathbf{w}(m+1) = \mathbf{w}(m) + \mu \begin{cases} (y_R(m)(R_R - y_R^2(m))) \\ + j.y_I(m)(R_I - y_I^2(m)) \end{cases} \mathbf{.u}^*(m)$$
(23)

In this paper, we propose the S-MMA for using in the OFDM applications which employing QAM signals. The S-MMA cost function satisfies a number of desirable properties, including multiple-modulus, symmetry, and (almost) uniformity. The S-MMA cost function exhibits a much lower misadjustment compared to CMA and MMA [8]. The proposed S-MMA algorithm is devised by embedding the sliced symbols in the dispersion constants [8]. The S-MMA cost function is

$$J = E \begin{cases} \left( y_R^2(m) - |\hat{x}_R(m)|^c R_R \right)^2 \\ + \left( y_I^2(m) - |\hat{x}_I(m)|^c R_I \right) \end{cases}^2$$
(24)

where x(m) is the predicted symbol and c is a positive constant. The S-MMA tap updating is  $\mathbf{w}(m+1) = \mathbf{w}(m)$ 

$$+ \mu \begin{cases} (y_{R}(m) || \hat{x}_{R}(m) |^{c} R_{R} - y_{R}^{2}(m)) \\ + j.y_{I}(m) || \hat{x}_{I}(m) |^{c} R_{I} - y_{I}^{2}(m)) \end{cases} \cdot \mathbf{u}^{*}(m)$$
(25)

The S-MMA update mechanism is aware of the dispersion of y(n) away from the closest symbol  $\hat{x}(m)$  in some statistical sense. The performance of an equalization algorithm maybe measured as the bit error rate (BER), the convergence rate and the residual ISI[8]. In this paper, the performance is measured by BER criteria.

### 6. Simulation Results

In this work, three tap updating algorithms (well-known LMS, MMA and S-MMA) are applied to the OFDM multicarrier modulation. For comparison, six standard channels, SUI[1] through SUI[6], with length of L=18 taps and AWGN (with mean=0) noise are employed. For transmission efficiency, the CP length was set to be smaller than the channel length i.d. v=16, and

hence the system had ISI impairment and AWGN noise simultaneously.

For each experiment, the BER is obtained from the ensemble average of 1000 independent Monte Carlo experiments. Because the results for SUI[1] through SUI[6] channels are almost the same, we have shown the results only for SUI[1] in Fig. 7. It is clear that the S-MMA has a much lower BER than the LMS and MMA algorithms, especially for high SNR's.



Fig.7. The effects of S-MMA on SUI[1] channel with AWGN noise

### 7. Conclusions

In this work. the S-MMA adaptive equalization was introduced for OFDM applications. The performance of the MMA and S-MMA was contrasted against the well-known LMS equalization in per-tone equalizer for SUI channels with the AWGN noise. For transmission efficiency, the length of the CP was set to be smaller than the channel length, and hence the system had simultaneous ISI impairment. Both analysis and simulations results show the gains and clearly verify that the S-MMA equalization, with an insufficient length of CP, has a lower BER than the most well-known LMS across all channel SNR's. Thus, the S-MMA equalization is a suitable candidate to replace for the LMS equalization in the OFDM modulation.

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